

# Cancellation of atmospheric turbulence angle-of-arrival fluctuations

Marcelo V. da Cunha Pereira,\* Luísa A. P. Filpi, and C. H. Monken

*Departamento de Física, Universidade Federal de Minas Gerais,  
Caixa Postal 702, Belo Horizonte, MG 30123-970, Brazil*

(Dated: February 16, 2012)

By using a turbulence chamber which emulates the dynamic behavior of the actual turbulent atmosphere, we show that the most deleterious wavefront aberration imparted to an optical beam, the angle-of-arrival fluctuation, which corresponds in practical situations to more than 87% of all the wavefront aberrations in free-space propagation, can be cancelled using two-photon correlation measurements and passive linear optics.

PACS numbers: 42.68 Ay, 42.50 Ex, 03.67 Hk, 42.25 Dd

## I. INTRODUCTION

Usually, when light is sent through free-space links, information is encoded in polarization because clear-air propagation disturbances don't affect it much [1] but, because it is a two-dimensional degree of freedom, the amount of information which can be transmitted per photon is limited. Photons spatial degrees of freedom allows for much larger dimensionality space to be accessed [2–6], the problem being that information encoded in these are typically very degraded by the atmosphere [7].

As has been shown recently in [8], the strong correlations in entangled states can assist in performing classical communication under the effect of specific sources of error. In this letter we report an open possibility of encoding information in the spatial degrees of freedom of a two-photon beam in a way to make it resilient against atmospheric turbulence or, more specifically, wavefront tilt caused by atmospheric inhomogeneities. Among the different phase aberrations imparted by the refractive index fluctuations in turbulent atmosphere, this has long been known [9] for being the most deleterious in terms of loss in resolution when long-exposure imaging is used [10], accounting for as much as 87% of the total wavefront distortion.

This aberration is routinely compensated by means of adaptive optics, where actuators change the shape of a telescope depending on a probe feedback [11]. The difference between this and our approach is that ours use the inherent correlation of the twin photons in such a way that no active compensation is required. We also don't require a bright source as in schemes that employ optical phase conjugation [12].

## II. THEORY

We consider a situation in which a beam of wavelength propagates in the  $z$  direction for a distance  $D$ , over which it suffers the effects of random refractive index fluctuations, and then is focused by a positive lens of focal

length  $f$  onto a photodetector. The beam may come from a laser, in which case we'll be interested in its intensity at the lens focal plane, or from a spontaneous parametric down-conversion (SPDC) process, where a nonlinear crystal is pumped by a well collimated laser beam of wavevector  $k_p$ . For the latter situation, we'll measure the fourth-order correlation by using coincidence measurements and the entity that behaves as a beam is the correlation of the photon pairs itself. The key result in this letter is to show that when dealing with SPDC, a proper preparation of the down-converted photons, through a passive and linear setup, can mitigate the effects of wavefront tilt. Transverse coordinates in the lens plane are denoted by  $\boldsymbol{\rho}$  while those in the plane of detection are  $\mathbf{u}$ , with eventual subindexes 1, 2 labeling different electromagnetic modes.

The refractive index fluctuations can induce both amplitude and phase fluctuations in the beam, the latter being the one in which we are interested here; in particular, wavefront tilt. It is described by an extra multiplicative factor of  $\exp[ik\boldsymbol{\theta}_j \cdot \boldsymbol{\rho}_j]$  in each of the field mode operators, where  $\boldsymbol{\theta}$  is a random variable denoting the tilt angle. This leads to the following two-photon wavefunction in the plane of the lens [6]:

$$A(\rho_1, \rho_2) = \tilde{E}\left(\frac{\rho_1 + \rho_2}{2}\right) \tilde{V}(\rho_1 - \rho_2) \times e^{i\frac{k_p}{2}\boldsymbol{\theta}_1 \cdot \boldsymbol{\rho}_1} e^{i\frac{k_p}{2}\boldsymbol{\theta}_2 \cdot \boldsymbol{\rho}_2}, \quad (1)$$

where  $\tilde{E}(\boldsymbol{\rho})$  is the transverse profile of the pump in the lens plane in the absence of turbulence and  $\tilde{V}(\boldsymbol{\rho})$  is the propagated phase-matching function, whose Fourier transform will be given below. Now, in the focal plane of the lens, the field amplitude is the Fourier transform of  $A(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ , from variables  $\boldsymbol{\rho}_j$  to  $\mathbf{q}_j = \frac{k_p}{2f}\mathbf{u}_j$  ( $j = 1, 2$ ). The linearity of the extra phase factor in  $\boldsymbol{\rho}$  means that the resulting transform can be put in terms of only the variables  $\mathbf{q}_+ = \mathbf{q}_1 + \mathbf{q}_2$  and  $\mathbf{q}_- = \frac{\mathbf{q}_1 - \mathbf{q}_2}{2}$ :

$$g_+(\mathbf{q}_1, \mathbf{q}_2) \propto E(\mathbf{q}_+ + k_p[\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2]) V\left(\mathbf{q}_- + k_p\frac{\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2}{2}\right) \approx E(\mathbf{q}_+ + k_p[\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2]). \quad (2)$$

In the above expression,  $V(\mathbf{Q})$  is the function

\* Electronic address: marcelop@fisica.ufmg.br

$\text{sinc}\left(\frac{L}{k_p n_p} |\mathbf{Q}|^2\right)$  propagated from the crystal's output face up to the lens plane and also the Fourier transform of  $\tilde{V}$  up to some phase factor [6]. In the thin crystal regime, it varies much slower with its argument than  $E$  and for this reason it was considered to be a constant.

If we get one of the photons of a pair (say, photon 2) to suffer an inversion in transverse coordinates  $\rho_2 \rightarrow -\rho_2$ , the two-photon amplitude function in the focal plane of the lens will be, instead,

$$g_-(\mathbf{q}_1, \mathbf{q}_2) \propto E\left(\mathbf{q}_+ + k_p \frac{\theta_1 - \theta_2}{2}\right). \quad (3)$$

Whatever the case, the coincidences rate is given by  $R_C = \langle |g|^2 \rangle$ , with the average being taken over the turbulent air variables. It can be seen that if  $\theta_1$  and  $\theta_2$  are correlated, cancellation will occur in  $g_-$  (while the turbulence effects on  $g_+$  will be the same as those on a laser beam with wavevector  $2k_p$ ). To enforce this, entangled photons need to be transversally confined to an area of the order of  $\varrho_c^2$ , where  $\varrho_c$  is turbulence coherence length and gives the distance above which phase aberrations become uncorrelated. Of course this has to be maintained during the entire propagation, since the net distortion is, as opposed to what might be suggested by the adopted model, a result of accumulated small perturbations over the whole traversed length.

### III. EXPERIMENT

To emulate the conditions present in the actual atmosphere we adapt a model put forward by Keskin [13] and depicted in figure 1: it is an aluminum box inside which air of different temperatures are mixed, which results in a random temperature field leading to an index of refraction that varies randomly in space and time [14]. Air is blown into the chamber by two fans, one of which has a set of resistors ( $6.4 \Omega$ ) in front of it. The air going through the resistors gets heated up and proceeds to the center of the chamber where it mixes with the air blown by the other fan, which is at room temperature. The optical beam goes in and out of the chamber through two oppositely positioned holes, propagating perpendicularly to the direction of the fans and through the center where mixing takes place. Two other fans work as exhausts to the mixed air in order to ensure the stationarity of the process.

Our source is a 5 mm long BiBO crystal cut for type-I (it generates photons with same polarizations) collinear SPDC (NL Crystal in figure 2). It is pumped by a cw He-Cd laser (wavelength 325 nm, power 42 mW), generating entangled photons the polarizations of which are rotated by a half-wave plate to  $45^\circ$  relative to the horizontal plane. Some of the pairs will have their component photons leaving at different directions off the subsequent polarizing beam-splitter (PBS) and will go through different arms of an interferometer before going into the

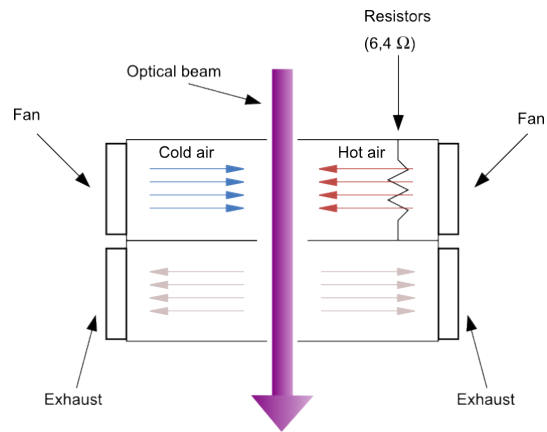


FIG. 1. (color online). A turbulence chamber that emulates the turbulent mixing of air at different temperatures which occurs in the atmosphere.

turbulence chamber. Because of another PBS right before the detectors, only these pairs will give rise to coincidence counts; pairs whose photons go through the same arm of the interferometer don't give rise to coincidences and are thus discarded. Inside the interferometer a half and a quarter-wave plates are placed oriented at  $45^\circ$  relative to the horizontal plane, so that a photon that was initially reflected will be transmitted by the PBSs and vice-versa. This results in all sampled photon pairs leaving through the same port of the second PBS, but with one photon in each pair suffering an additional reflection, thus effecting the inversion  $\rho_2 \rightarrow -\rho_2$ . After collinearly propagating through the turbulence chamber, they are collected by a lens of focal length 50 cm that focuses them on the detection planes. The detectors are Perkin Elmer SPCM-AQR-14 photon counting modules in front of which we place 10 nm bandpass filters centered at 650 nm and 50  $\mu\text{m}$  wide slits. The slits, each having one of its dimensions much larger than the transverse widths of the beam, effectively average out the effects pertaining to the  $y$  direction, leaving as parsed only the  $x$  direction, along which the inversion scheme is implemented.

In our experiment, a laser beam with a gaussian profile pumps the nonlinear crystal leading to an also gaussian coincidence profile (see equations (2) and (3)). We measure the curve with 20s for each point and perform a gaussian fit, repeating the process for different turbulence strengths. This can be varied by altering the input voltage in the resistors. The widths of the fitted curves are then registered and plotted against the square root of the Rytov variance  $\sqrt{\sigma_R^2}$ , a conventional figure of merit for the turbulence strength. The measurement is repeated by removing the interferometer and placing a half-wave plate just before the last PBS, to ensure coincidence counts. These two sets of measurements correspond to the  $g_-$  and  $g_+$  cases, respectively, and are shown together in figure 3. Another measurement is performed using a cw He-Ne laser beam propagating under the same conditions, and the collected data is also included in the same plot. One can see smaller broadening in the  $g_-$  case

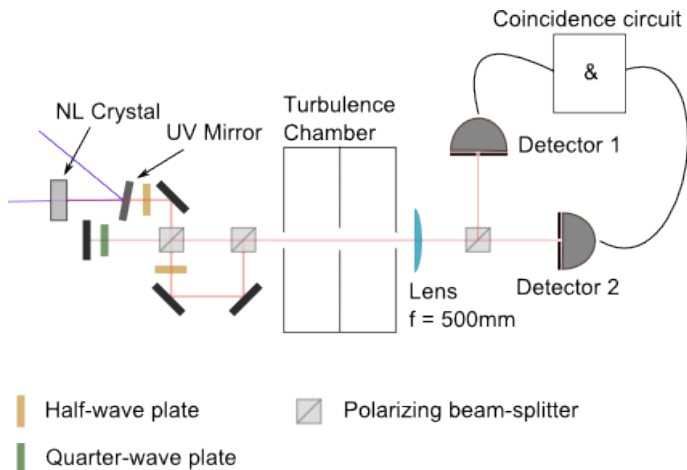


FIG. 2. (color online). Schematics of the experimental setup. The interferometer following the entangled photons source (NL Crystal) ensures that one of the photons suffers an additional reflection relative to the other.

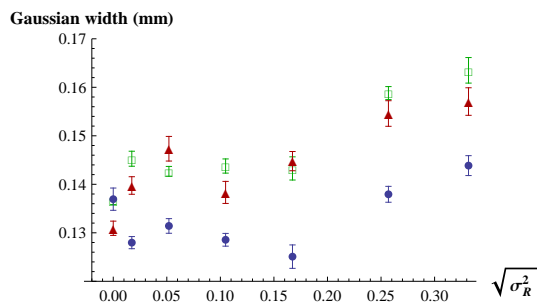


FIG. 3. (color online). Gaussian widths of the measured profiles as a function of  $\sqrt{\sigma_R^2}$  (see text). Blue circles and red triangles are for coincidence measurements with and without the interferometer, and green empty squares is for the intensity measurements of a He-Ne laser beam.

as well as similar broadening in  $g_+$  and laser beam data, as predicted.

It is, however, to be kept in mind that wavefront tilt is only but one of the phase aberrations imparted to the beam and that the next relevant ones, the quadratic aberrations, are not cancelled in the present scheme, and indeed start showing up at higher voltages. Their importance are, however, overestimated in this experiment because of the limited dimensions of the turbulence chamber. A more complete description of the turbulent air effects has to take into account its size and, as shown in [15], a consequence of this confinement is a shift of the relative importance of the degrading effects from wavefront tilt towards those of higher-order aberrations.

Calibration of the turbulence chamber was made by sending an expanded He-Ne laser beam through it, and focusing onto a CCD camera. Using the known beam radius  $w = 68\mu\text{m}$  and traversed path length inside the chamber  $D = 76\text{ cm}$ , and measuring the widths  $\sigma(P)$  at the focal plane as a function of power  $P$  dissipated by the resistors, the Rytov variance was computed with the

following formula [16]:

$$\sigma_R^2 = \left( \frac{\sigma^2}{\sigma_0^2} - 1 \right) \left( 1.33\Lambda^{5/6} \right)^{-1} \quad (4)$$

, where  $\Lambda = \frac{2D}{k\omega^2}$  and  $\sigma_0 = \sigma(0)$ . At the highest power we get a Rytov variance of  $\sigma_R^2 = 0.17$ , close to the value 0.3 that conventionally separates weak and strong turbulence regimes [1]. It is an established fact that the theory describing phase fluctuations in turbulence has its validity extending far into the strong regime, so that the cancellation exposed here should be expected also under this circumstance [14].

#### IV. DISCUSSION

The cancellation can be understood in terms of Klyshko “advanced wave” picture [17], a formal equivalence between coincidence detection of down-converted photons and the detection of a photon coming from one of the detectors, reflecting at the non-linear crystal and propagating back to the other detector. In the latter scenario the cancellation can be seen as follows: the photon leaves the first detector, has its direction changed by the turbulence, inverts the deflection angle at the interferometer and then get it undone upon propagating again through the same inhomogeneities inside the chamber before reaching the second detector.

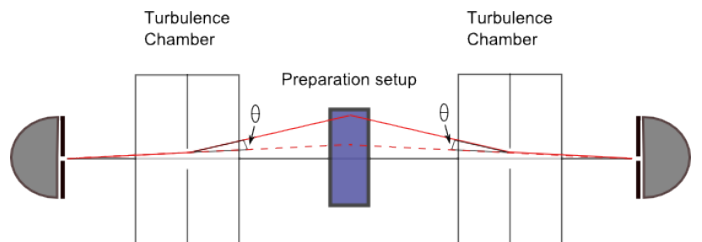


FIG. 4. (color online). Unfolded “Klyshko” picture of the cancellation process. Photon has turbulence deflection inverted in the preparation setup and then undone when going again through the chamber. Solid line shows path with deflection by turbulence inhomogeneities and dashed line shows path without deflection.

An interesting question arises as whereas entanglement is actually necessary to attain cancellation: in fact what was needed here was a  $u_1 + u_2$  dependence on the detectors’ positions  $u_1$  and  $u_2$ , the kind of which appears on the conditional fringes of Greenberger et al. [18]. In [19] it is shown that for a known transfer function, classical correlations can be engineered in such a way as to reproduce any joint detection probability attainable in a single plane with entangled photons. Here, we only make use of the far-field correlations, but the transfer function is random and unknown, and it is not clear if in this case one can reproduce the obtained results with only classical correlations and a passive setup.

Sending unspoiled entangled photons, however, is an advantage in itself because it allows for the use of quantum communication protocols such as super-dense coding and entanglement purification. Investigations on the behavior of spatial entanglement under atmospheric turbulence has been already pursued but the question still remains as to whereas one can effectively protect it by properly preparing the states to be sent. This is to be addressed in future research.

It should also be stressed out that the tilt cancellation can also be verified theoretically by using the generalized Huygens-Fresnel method, which includes in the propagators the effect of turbulence [20]. This method have been

used somewhat extensively together with an approximation usually only employed to make calculations more tractable, which is assuming a square scaling-law for the turbulence structure functions instead of the more correct Kolmogorov 5/3 law. As shown in [21], it happens that this approximation means physically that only angle-of-arrival aberrations are taken into account.

## V. ACKNOWLEDGEMENTS

Funding provided by brazilian agencies CNPq and FAPEMIG.

- 
- [1] R. Fante, Proceedings of the IEEE **63**, 1669 (1975).
  - [2] G. Molina-Terriza, J. P. Torres, and L. Torner, Phys. Rev. Lett. **88**, 013601 (2001).
  - [3] G. Gibson, J. Courtial, M. Padgett, M. Vasnetsov, V. Pas'ko, S. Barnett, and S. Franke-Arnold, Opt. Express **12**, 5448 (2004).
  - [4] L. Neves, G. Lima, J. G. Aguirre Gómez, C. H. Monken, C. Saavedra, and S. Pádua, Phys. Rev. Lett. **94**, 100501 (2005).
  - [5] J. B. Pors, S. S. R. Oemrawsingh, A. Aiello, M. P. van Exter, E. R. Eliel, G. W. 't Hooft, and J. P. Woerdman, Phys. Rev. Lett. **101**, 120502 (2008).
  - [6] S. Walborn, C. Monken, S. Pádua, and P. Souto Ribeiro, Physics Reports **495**, 87 (2010).
  - [7] C. Paterson, Phys. Rev. Lett. **94**, 153901 (2005).
  - [8] R. Prevedel, Y. Lu, W. Matthews, R. Kaltenbaek, and K. J. Resch, Phys. Rev. Lett. **106**, 110505 (2011).
  - [9] D. L. Fried, J. Opt. Soc. Am. **55**, 1427 (1965).
  - [10] D. L. Fried, J. Opt. Soc. Am. **56**, 1372 (1966).
  - [11] J. W. Hardy, Scientific American **270**, 60 (1994).
  - [12] D. S. Simon and A. V. Sergienko, (2011), arXiv:1105.0128v2.
  - [13] O. Keskin, L. Jolissaint, and C. Bradley, Appl. Opt. **45**, 4888 (2006).
  - [14] J. W. Strohbehn, S. F. Clifford, M. E. Gracheva, A. S. Gurvich, S. S. Kashkarov, V. V. Pokasov, A. Ishimaru, J. H. Shapiro, J. L. Walsh, and P. B. U. and, *Laser Beam Propagation in the Atmosphere*, edited by J. W. Strohbehn, Topics in Applied Physics, Vol. 25 (Springer-Verlag, 1978) p. 321.
  - [15] D. M. Winker, J. Opt. Soc. Am. A **8**, 1568 (1991).
  - [16] L. C. Andrews and R. L. Phillips, *Laser beam propagation through random media*, edited by SPIE (SPIE press, 2005) p. 782.
  - [17] D. Klyshko, Physics Letters A **132**, 299 (1988).
  - [18] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Physics Today **46**, 22 (1993).
  - [19] R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. **89**, 113601 (2002).
  - [20] R. Fante, in *Progress in optics*, Vol. 22, edited by E. Wolf (Elsevier, 1985) pp. 341–398.
  - [21] S. M. Wandzura, J. Opt. Soc. Am. **70**, 745 (1980).